3D microwave simulation for spherical tokamaks

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Acknowledgments:

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THE UNIVERSITY of York

Outline

1. Why study microwave interactions?
2. Underlying plasma physics
3. 3D full-wave simulations
4. Ongoing work

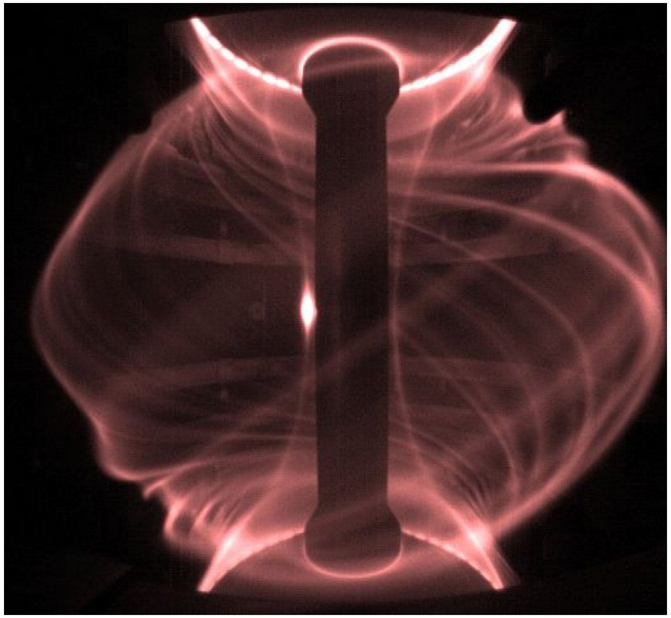
3D effects

• Spherical tokamak edge plasma contains 3D density fluctuations (filaments, blobs etc.) and magnetic shear

• Interactions with microwaves must be understood for EC emission diagnostics, heating and current drive (for EBW, the effect on mode conversion)

• 3D full-wave modelling necessary to explore interactions in detail, investigate new physics and aid interpretation of experimental data

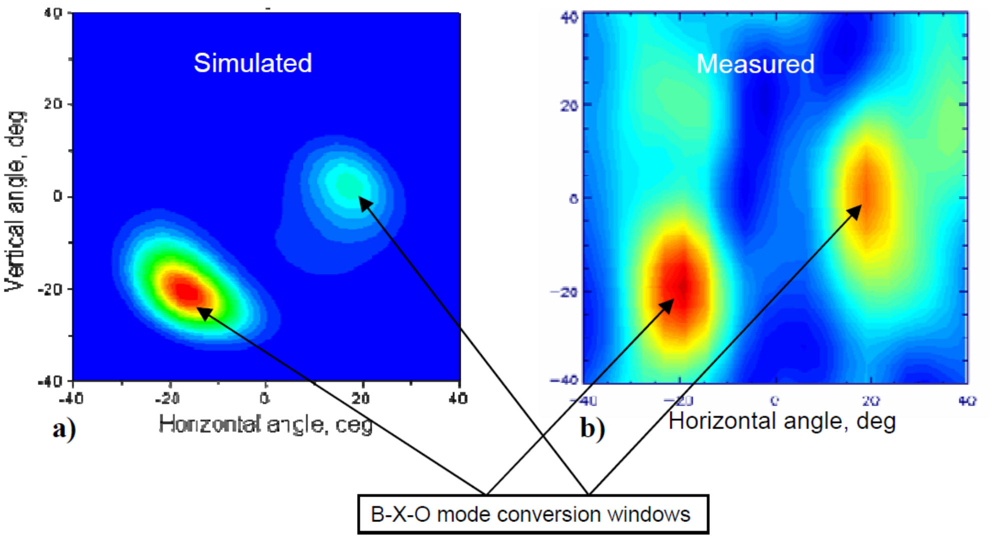
• Extrapolation beyond current experiments



MAST plasma showing filaments at edge

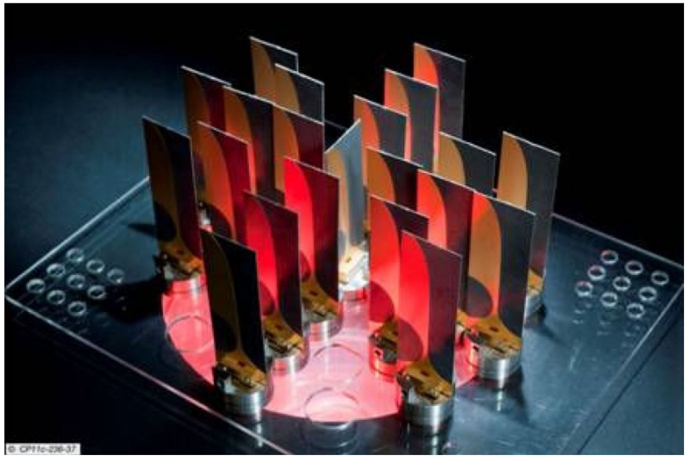
SAMI diagnostic

• York/Culham collaboration (*V. Shevchenko et al., 2012, arXiv: 1210.3278 [physics.plasm-ph]*)



• Images microwave emission at 10 - 35 GHz → radial range through edge. High time resolution (~10 μ s)

• In process of using this data to generate an edge **J**-profile – aim to reconstruct pedestal during inter-ELM period. Major H-mode issue



• Observed fluctuations much higher than expected! (Dave had a poster…)

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Cold plasma dispersion

- Assume plane waves & rewrite linearised Maxwell's equations using dielectric permittivity tensor . Matrix equation obtained:

$$\left[\mathbf{kk} - k^2 \underline{\underline{I}} + \frac{\omega^2}{c^2}\underline{\underline{\varepsilon}}\right] \cdot \mathbf{E} = \underline{\underline{M}} \cdot \mathbf{E} = 0$$

- Ideal, cold, magnetised plasma with uniform equilibrium $\mathbf{B_0}$-field. Evaluate dielectric tensor using linearised fluid equation for electrons:

$$m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\left(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}_0\right)$$

- Assume $\mathbf{B_0}$ and $\mathbf{k}$ are perpendicular (i.e. propagation $\perp$ background magnetic field) – find 2 solutions to matrix equation:

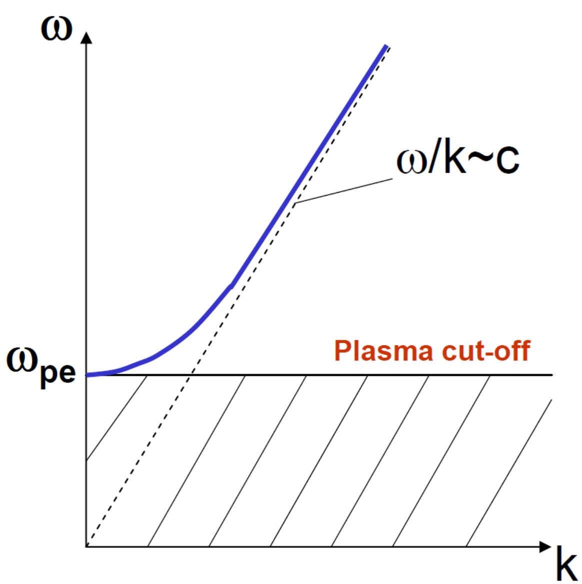
$$n^2 = 1 - X \qquad\qquad n^2 = 1 - \frac{X(1-X)}{1 - X - Y^2} \qquad \left(X = \frac{\omega_{pe}^2}{\omega^2}, Y = \frac{|\omega_{ce}|}{\omega}\right)$$

O-mode X-mode

Cold plasma modes

O-mode

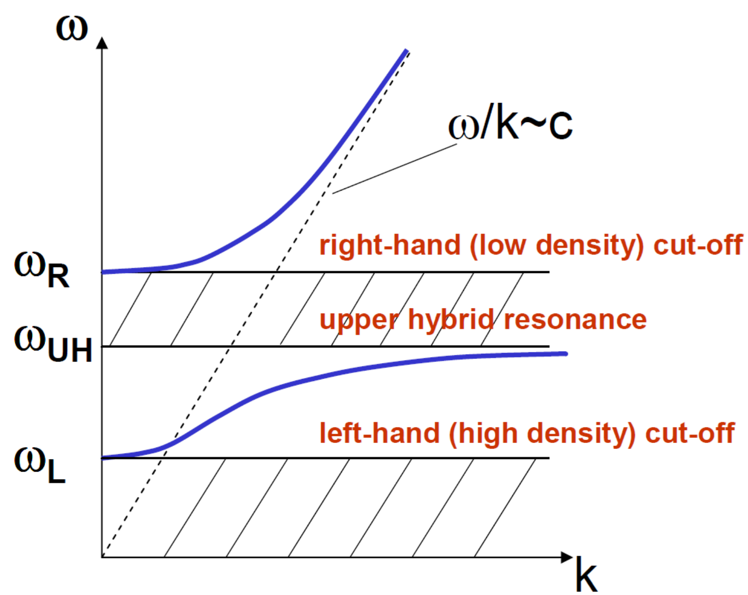
$$\mathbf{E_1} \parallel \mathbf{B_0}$$



V.F. Shevchenko "EBW in fusion plasma" lectures, 2009

X-mode

$$\mathbf{E_1} \perp \mathbf{B_0}$$



$$\omega_{UH} = \omega_{ce}^2 + \omega_{pe}^2$$

$$\omega_{R/L} = \frac{1}{2}\left(\sqrt{4\omega_{pe}^2 + \omega_{ce}^2} \pm \omega_{ce}\right)$$

X-O conversion

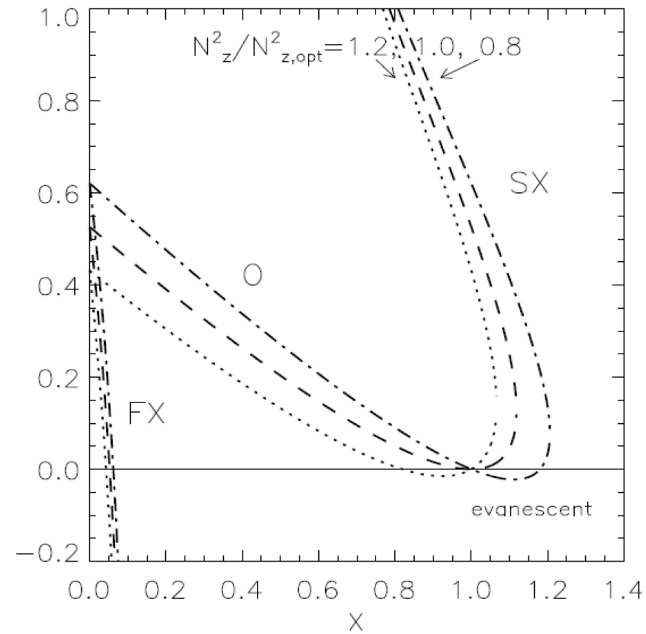
- Mode conversion from X-mode to O-mode (and vice versa) occurs at O-mode density cut-off ($\omega = \omega_{pe}$) if wave is obliquely incident at an optimal angle to $\mathbf{B_0}$

- At suboptimal angles, wave tunnels through evanescent layer of finite width (dependent on density scale length L_n), reducing conversion efficiency

- Using WKB approximation, this efficiency T was calculated by Mjølhus :

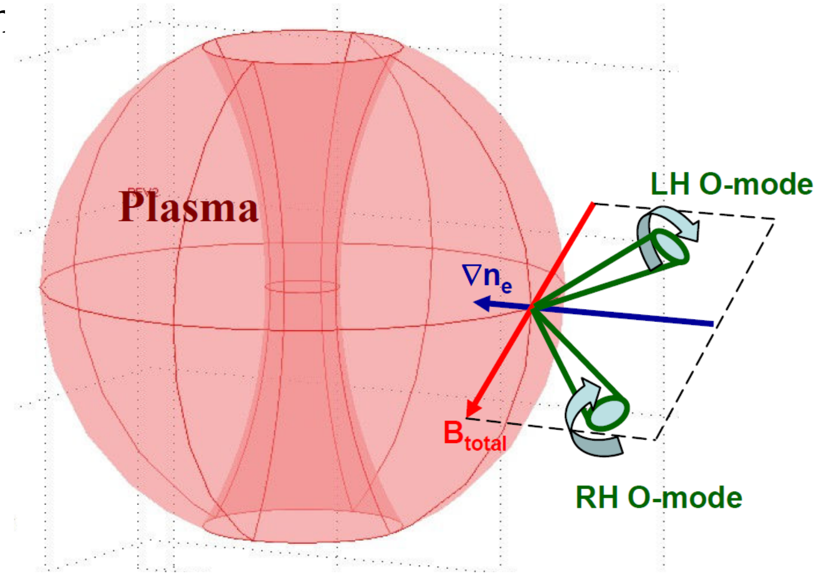
$$T = \exp\left\{-\pi k_0 L_n \sqrt{\frac{Y}{2}}\left[2(1+Y)(N_{z,opt}-N_z)^2 + N_y^2\right]\right\}$$

E. Mjølhus, J. Plasma Physics 31 (1) 7, 1984



B-X-O in a spherical tokamak

- Electrostatic electron Bernstein modes (EBWs) are excited near cyclotron resonances and couple to X-mode at the upper hybrid resonance
- Typically, in STs such as MAST running at high β (higher n_e, lower $\mathbf{B_0}$), $\omega_{pe} > \omega_{ce}$
- Problematic for conventional ECE diagnostics but allows B-X-O conversion to produce two cones of O-mode emission fror
- Cones emitted in the plane of $\mathbf{B_0}$ and ∇n_e
- ∇n_e known from TS diagnostic
- Imaging these cones gives pitch of $\mathbf{B_0}$ $\rightarrow \mathbf{B}_\theta \rightarrow \mathbf{J}$ at edge



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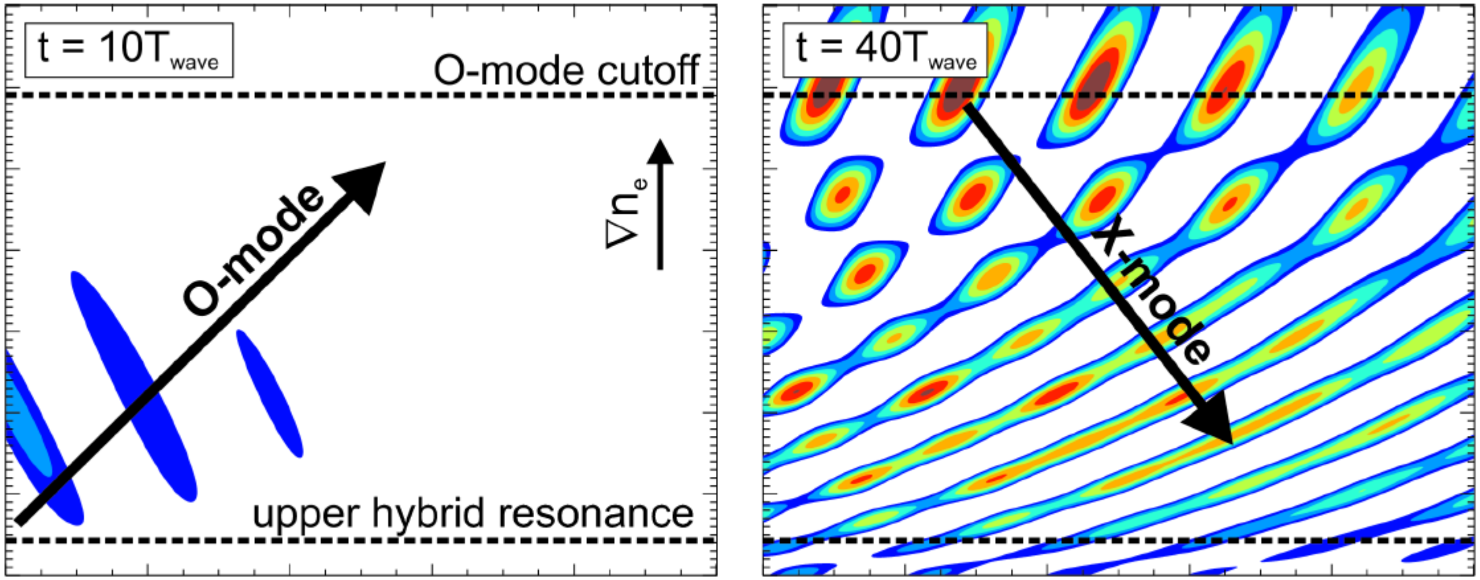
Previous modelling



A. Köhn, Ph.D. Thesis, 2010

• Ray/beam tracing applied to beam propagation problems, but fast variations in refractive index make it unsuitable for conversion region

• 2D full-wave modelling of O-X conversion by A. Köhn, using the code IPF-FDMC

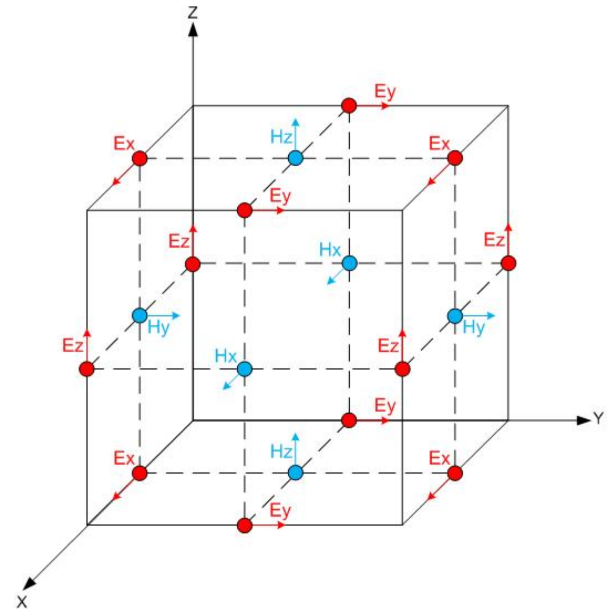
• Detailed insight into the mode conversion process

3D simulations – ADE-FDTD

- New code developed in support of experimental project

- 3D finite difference time-domain (FDTD) method for solving Maxwell's equations :

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0}\nabla \times \mathbf{E} \qquad \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon_0}\nabla \times \mathbf{H} - \frac{1}{\varepsilon_0}\mathbf{J}$$



- Discretise field components to staggered grid to simplify calculation of numerical curl

- Substitute in 2nd order centred difference formulae in both space and time

- Obtain leapfrog equations for updating E and B-fields

- For plasma dielectric response, solve: $\frac{\partial}{\partial t}\mathbf{J} = \omega_{pe}^2 \epsilon_0 \mathbf{E} - \frac{e}{m}\mathbf{J} \times \mathbf{B_0}$

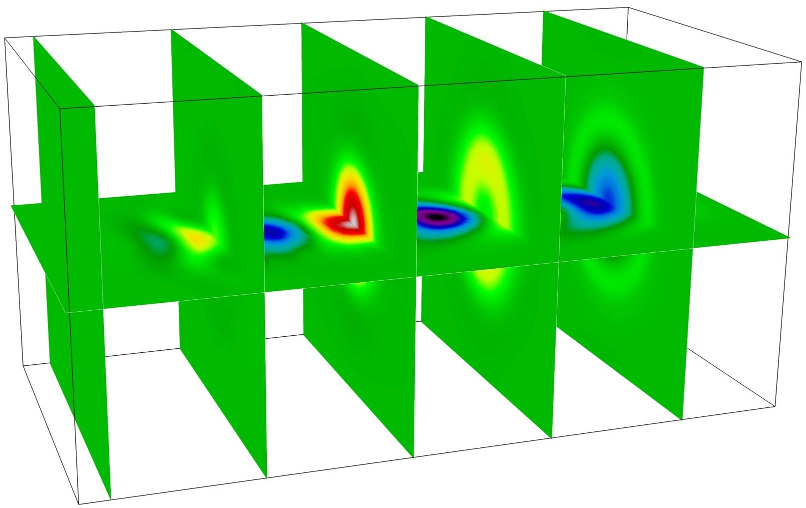
Code details

- Written in C++. Data-level parallelisation (spatial domain) using MPI

- Arbitrary static n_e and background $\mathbf{B_0}$ profiles specified – incident beam then excited using TF/SF source term and simulation run in time domain

- Perfectly matched layer (PML) boundary conditions: very thin absorbing regions

- Full 3D grid output at each timestep very large; virtual sensors reduce output dimensionality / sampling frequency. Transmission coefficients calculated in post-analysis



O-mode 3D Gaussian beam propagation

- Future development will include hot plasma terms in order to investigate full B-X-O conversion

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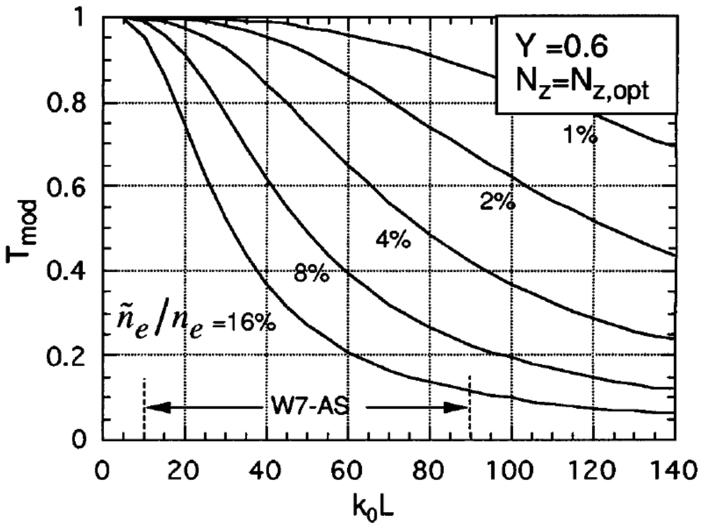
Density fluctuation studies

- Density fluctuations – beam diverges, reduction of conversion efficiency



H.P. Laqua et al, PRL 78 (18), 3467-3470 (1997)

- Analytic modelling - pdf approximating beam divergence used to modify Mjølhus formula

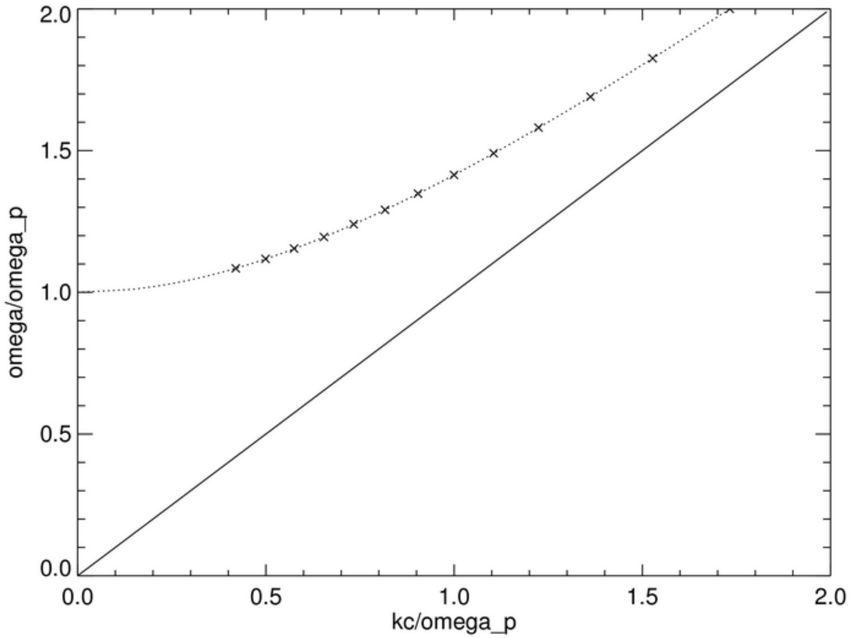
- Initial 1D and 2D full-wave modelling carried out by Köhn using IPF-FDMC

- 3D structures (filaments) at tokamak edge. For oblique incidence, problem is inherently 3D – 2D modelling is forced to choose single cut through profile

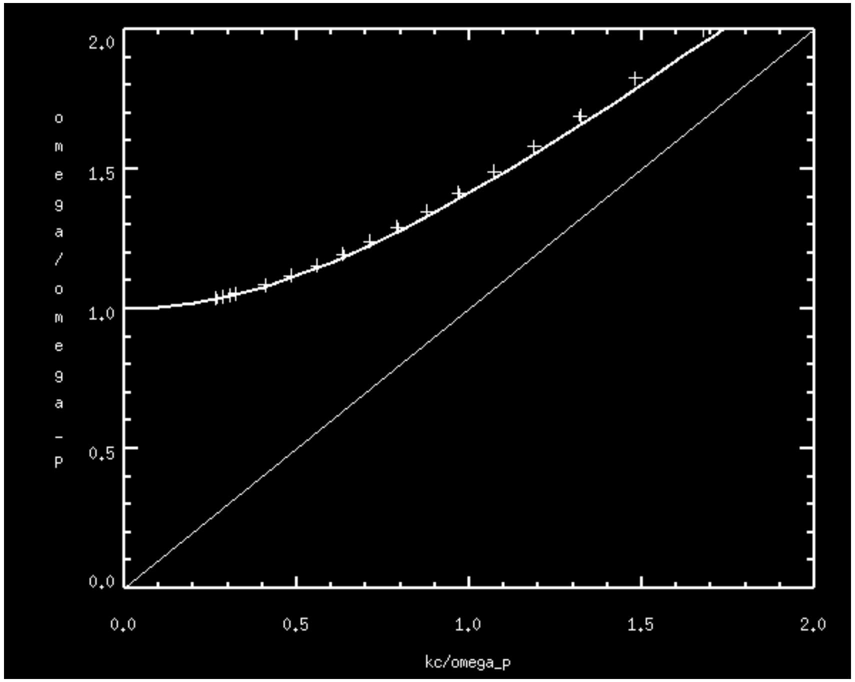
- Test validity of Laqua result across different regimes

Code comparison

- New 3-D code compared against results from IPF-FDMC.
- First stage – dispersion relations



3D code (Williams)

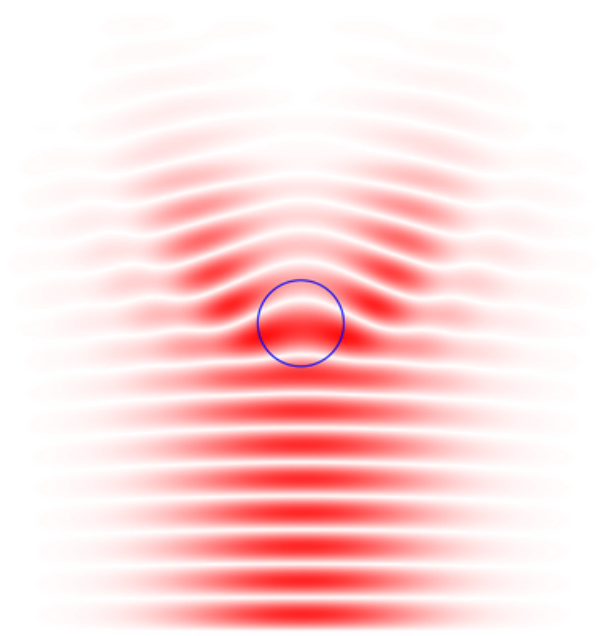


IPF-FDMC (Köhn)

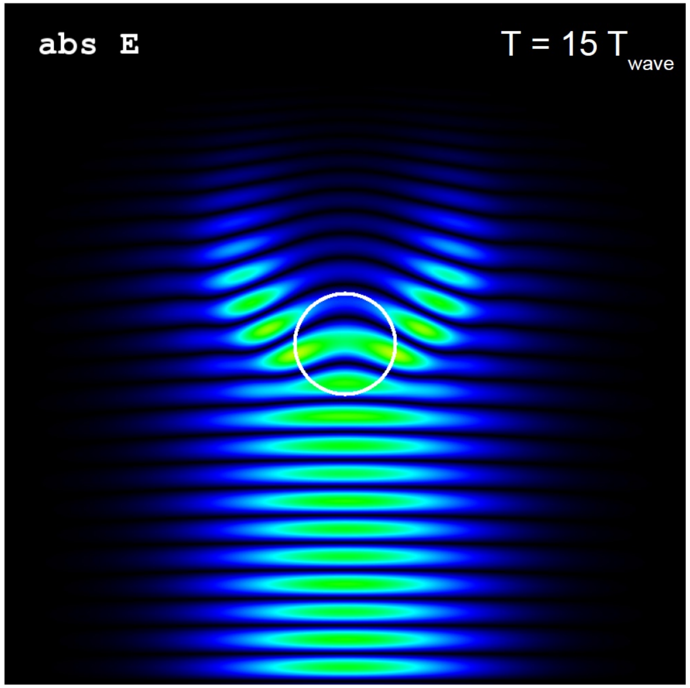
O-mode dispersion relation : dashed line – analytical, points : numerical

Code comparison (2)

• 2nd stage – add blob with Gaussian profile, peak density below critical, to homogeneous plasma background



3D code (Williams)

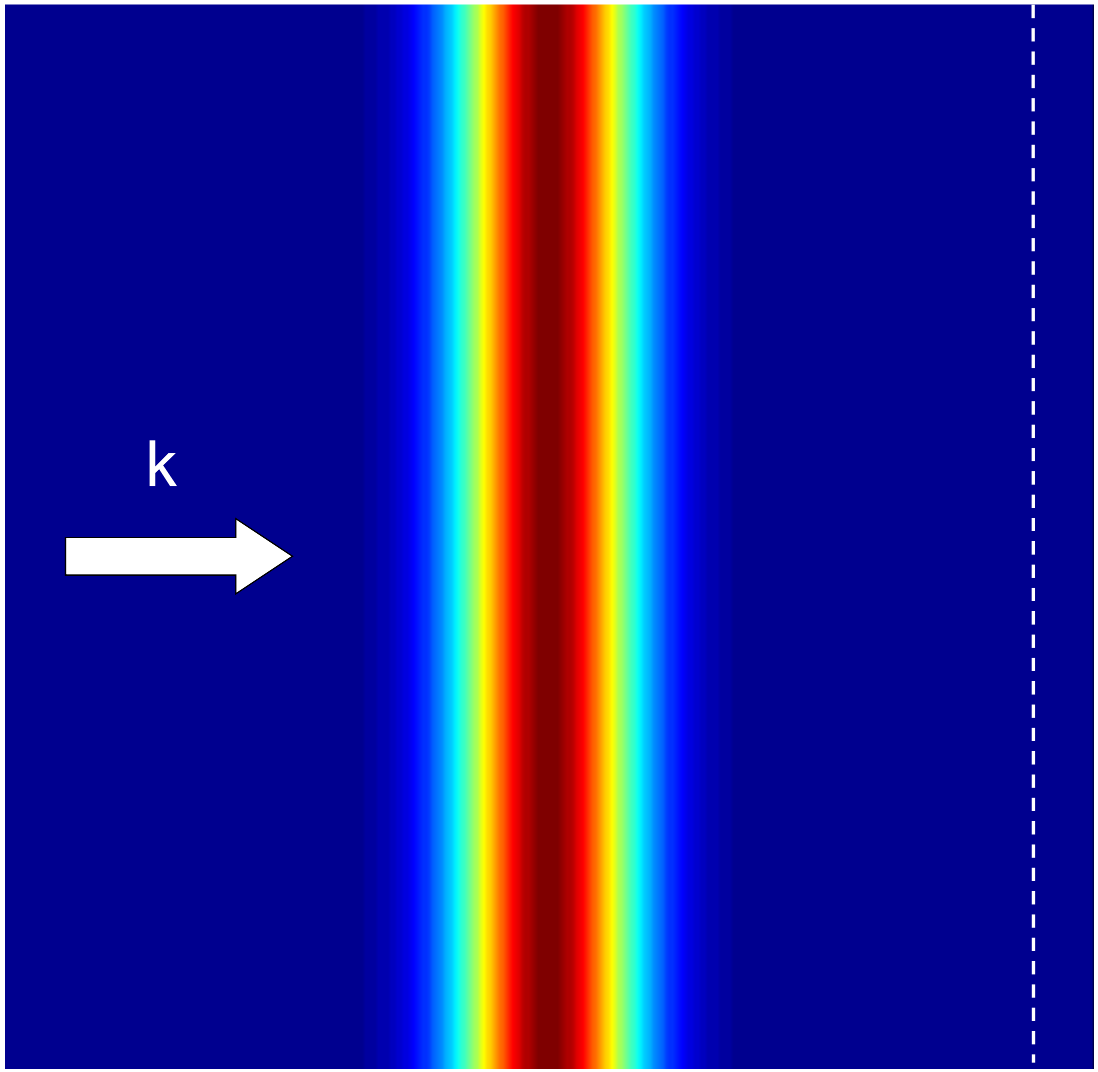


IPF-FDMC (Köhn)

• Circle at location of X = 0.6 surface. Codes agree on beam scattering

Filament scattering

Incident beam



Backplane

Analysis

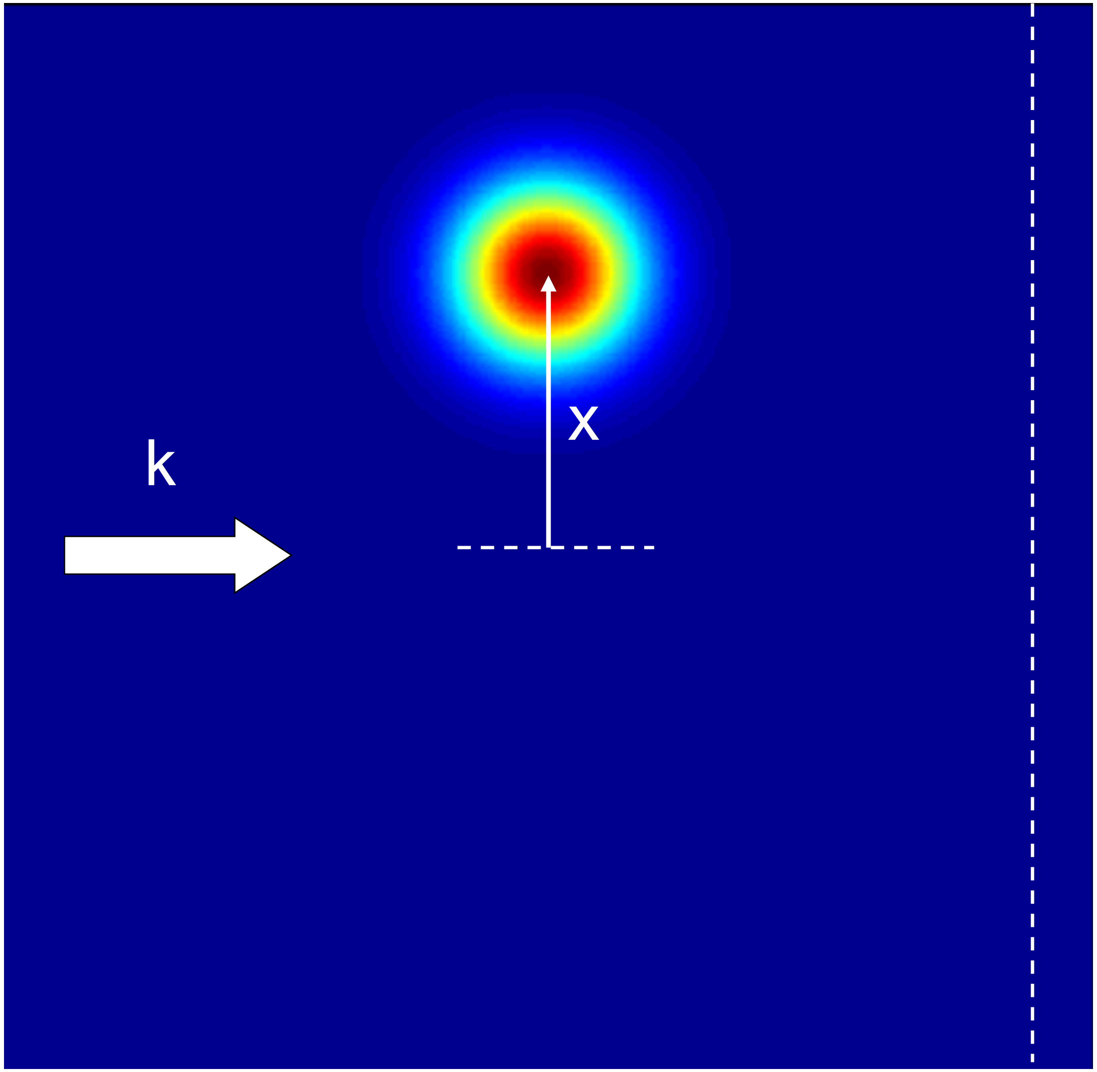
- Average electric field on backplane over several cycles
- Calculate total **E**, mean and σ in 2 dimensions – estimate of 'degree of scattering'

$$\mu = \frac{\int\int \mathbf{x} E_{avg}(x,y)\,dx\,dy}{E_{tot}}$$

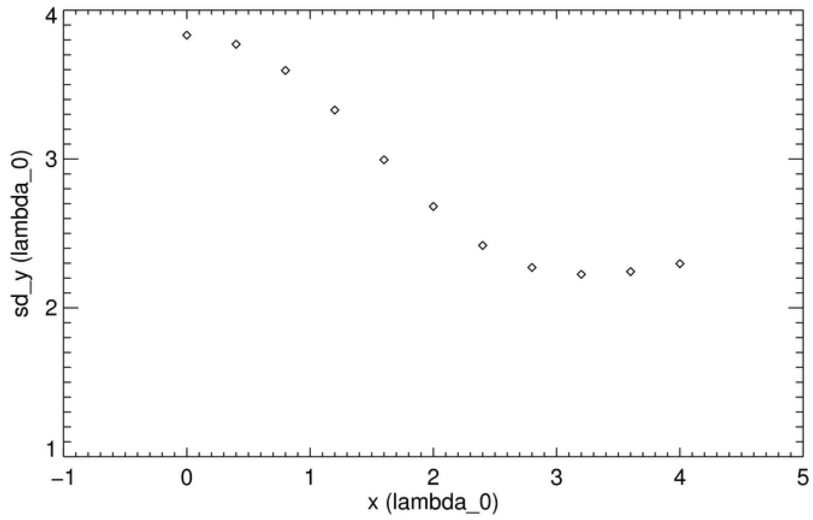
$$\sigma = \frac{\int\int (\mathbf{x}-\mu)^2 E_{avg}(x,y)\,dx\,dy}{E_{tot}}$$

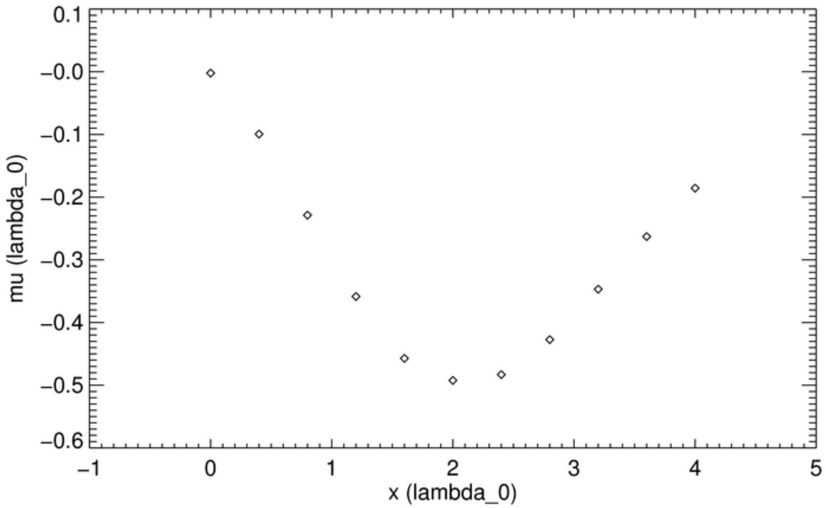
- Scan each parameter through experimentally relevant values

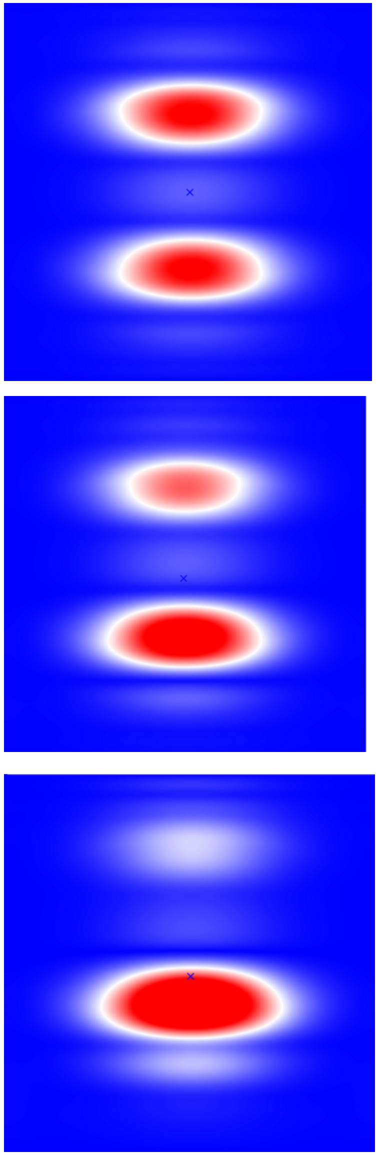
Parameter 1 : position



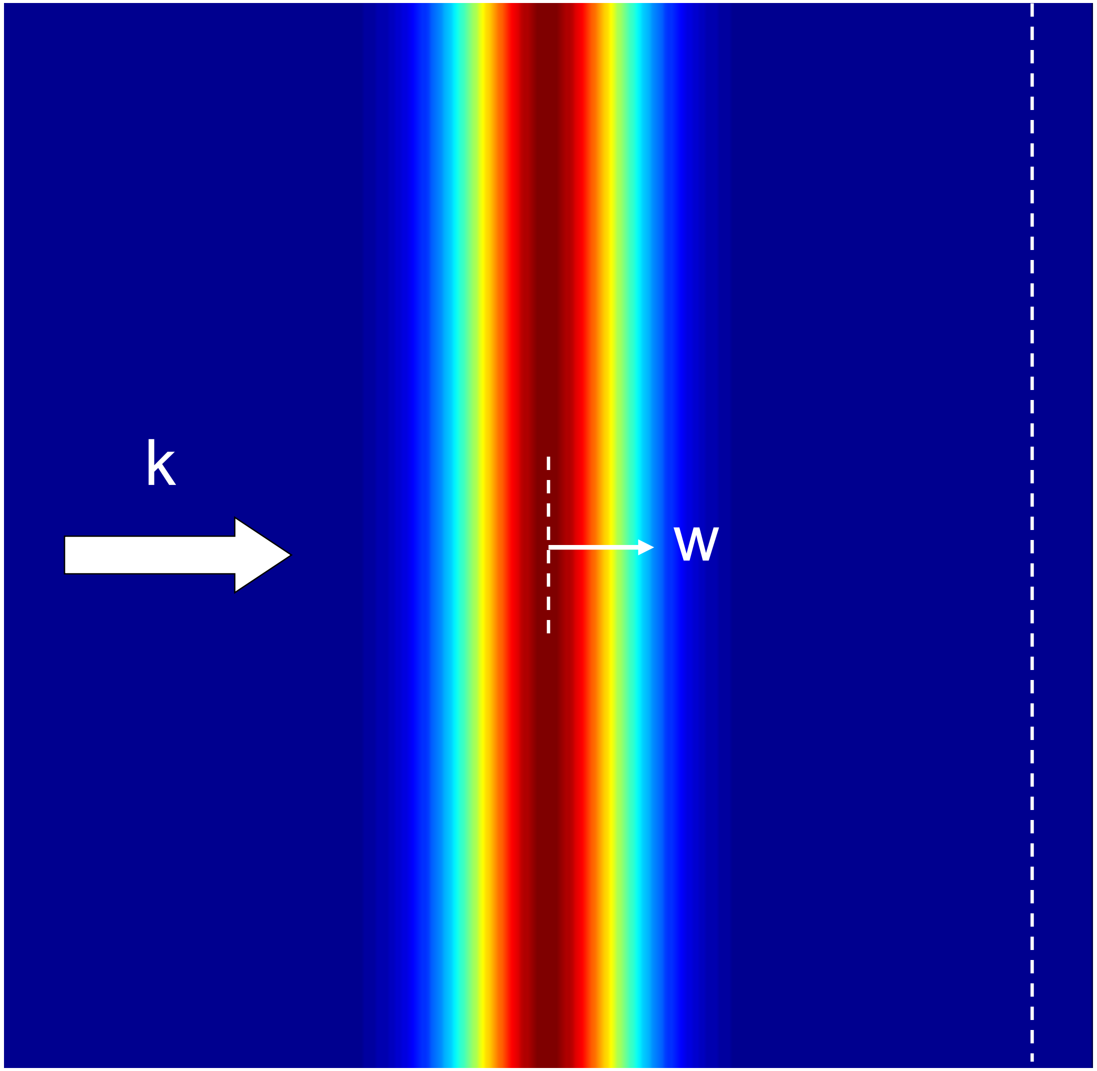
Position scan results



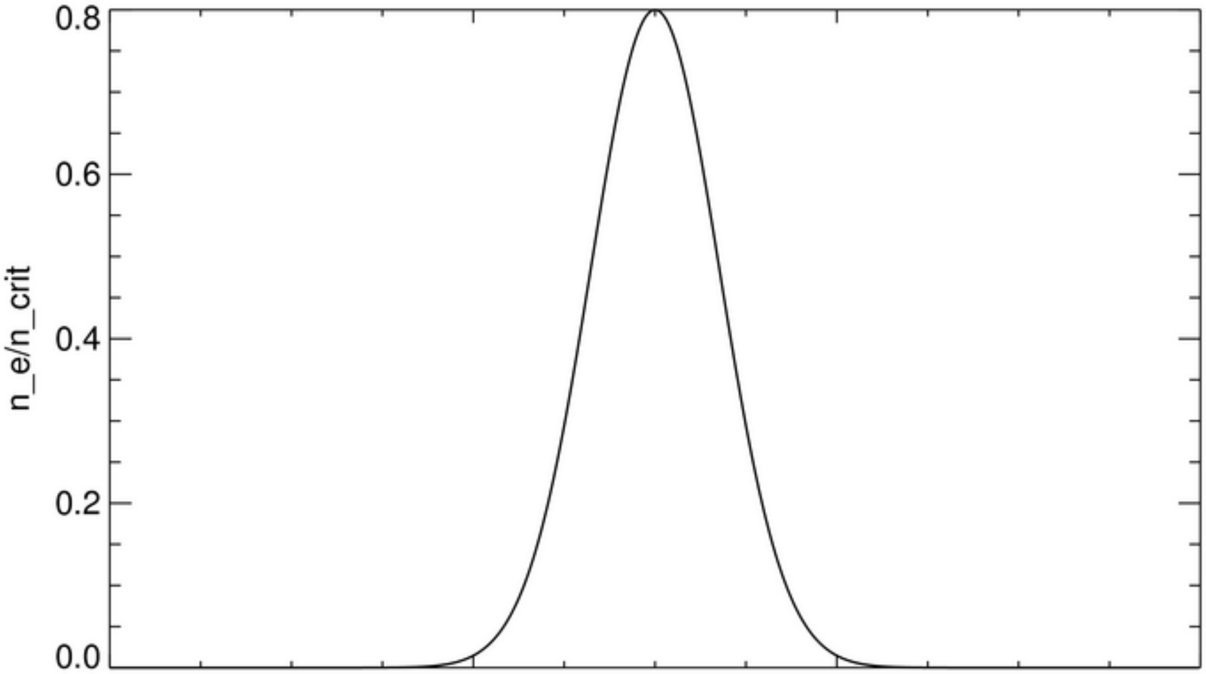




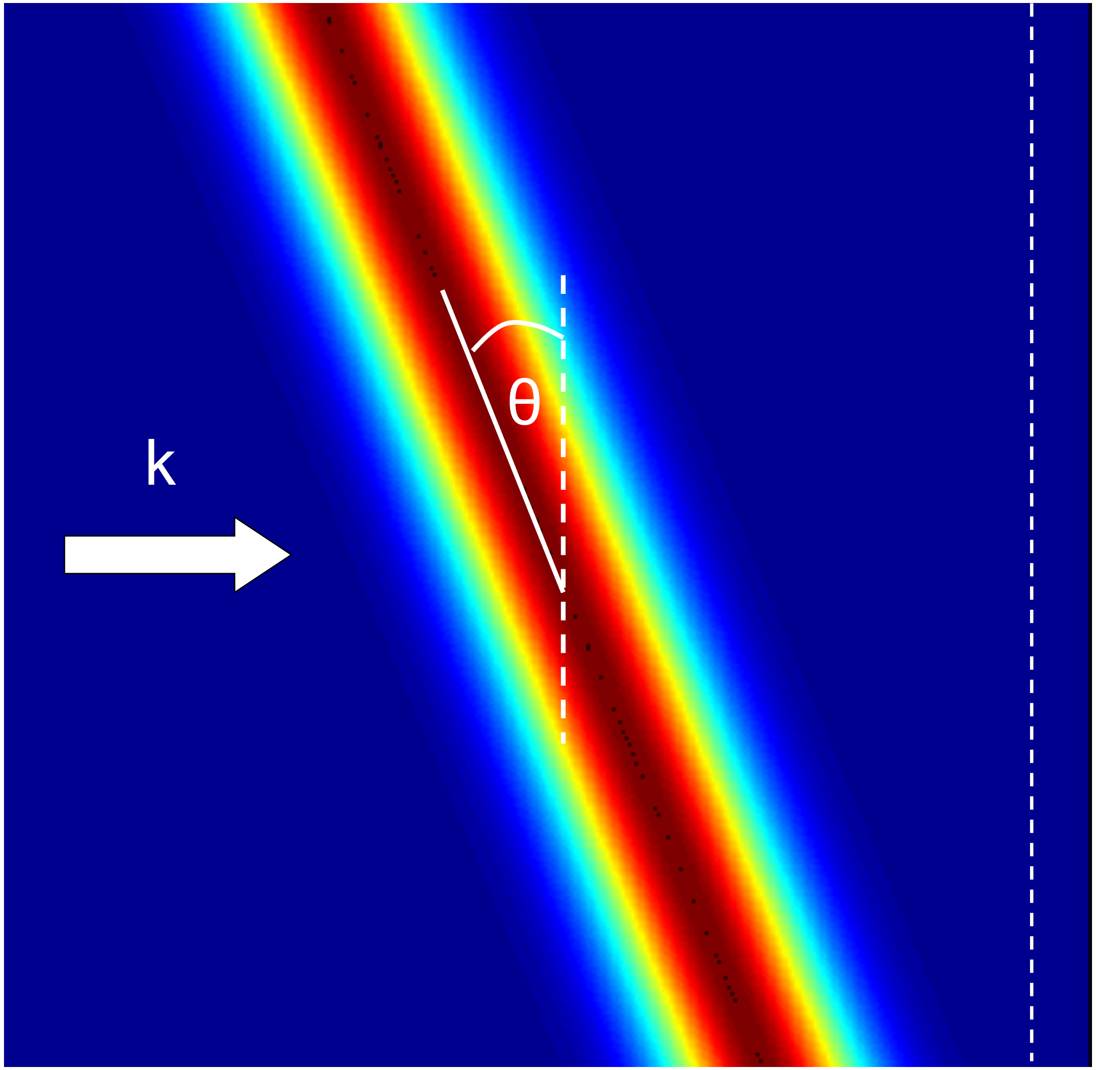
Parameter 2 : width



Parameter 3 : density



Parameter 4 : angle



Next steps

- Investigate the influence of presence of blob near mode conversion surface on mode conversion efficiency – compare against Laqua formula

- Include more realistic turbulent profiles for ST mode conversion region (generated from code e.g. GS2) for highest possible relevance to experimental studies. Average results over a set of perturbed profiles

- Include real MAST experimental profiles for comparison

- Investigate the effect of magnetic shear on mode conversion efficiency

- Can the effect of magnetic fluctuations be distinguished from that of density fluctuations?

Summary

• A new 3D FDTD code has been developed to simulate mode conversion in a fusion plasma.

• This is being used to investigate 3D effects including that of density fluctuations at the turbulent boundary of a spherical tokamak.

• These results are being compared against 2D simulations in a collaboration with IGVP Stuttgart.

• Results will aid the interpretation of data from new MAST diagnostic producing 2D images of mode conversion windows.

Thank you for listening.